Policy Making for Oil Imports by United Nations' Countries using a Multi-Objective Mixed Integer Linear Programming Approach

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Abstract

This paper highlights a method to assist policy making for oil imports for United Nations' member states to reduce their cost (including shipment) and pollution (including carbon dioxide and sulphur dioxide emissions), and assists with the transition towards United Nations Sustainable Development Goal Number 7, of being able to have a world with cleaner and affordable energy. Further, this paper discusses the use of Mixed Integer Linear Programming to solve a global optimal cost for oil transportation to ensure energy security, whilst reducing the cost and pollution to the environment. Techniques such as Multiple Linear Regression, Branch and Bound, and the use of Primal and Dual values are used to solve this optimization problem. Future work can be done to enhance solutions to this problem, consisting of certain aspects of Game Theory, such as the Nash equilibrium and shared utility to ensure that the difference between each cost of each country is minimal. This provides a solution which policy-makers will be amenable to implementing.

Keywords: Mixed integer linear programming; Policy making for united nations; Pollution; Oil imports and exports; Mathematical optimisation.

1 Background and Applications of Mixed Integer Linear Programming in the Field of Sustainability

Mixed Integer Linear Programming is a technique based on the concept of Linear Programming with added constraints of some integer variables and some non-integer variables. This technique is used to solve various different problems, such as scheduling problems, space optimisation problems, and the integration of renewable energy into energy grids. One example of the use of MILP is in technician scheduling and routing problems with many different constraints on tasks and skills, as discussed in [Methlouthi et al. 2016]. Such MILP formulations normally focus more on the usage of binary variables and are formulated based on physical constraints, such as working efficiency and time constraints. Another application is the optimisation of space to maximise the energy output from wind turbines, as discussed in [Kuo et al. 2016] and [Archer et al. 2011]. This focuses more on constraints regarding space and location rather than binary variables. Another application of optimisation of space using MILP includes the integration of different renewable energy sources to maximise energy production, as discussed in [Morais et al. 2010] and [Alberizzi et al. 2020]. This is similar to the integration of renewable energy into small communities, which is discussed in [Cosic et al. 2021].

Another aspect of sustainability which is solved using MILP is covered in [Kantor et al. 2020] and [Ren and Gao 2010], which discuss the application of MILP in the distribution of energy load and an integrated energy system.

This paper focuses on the aspect of energy integration into sustainability objectives. The goal is to make the world move toward a more sustainable path and align with the United Nations' Sustainable Development Goal Number 7: To have clean and affordable energy by 2030, as shown in Figure 1 below.

Figure 1: Sustainable Development Goal Number 7: Clean and Affordable Energy

2 Introduction

Due to the emergence of increasing global temperatures and the accelerated consequences of global warming, it is of utmost importance to reduce the amount of greenhouse gases being released through the burning of fossil fuels. Unfortunately, countries have not been able to harvest the full potential of renewable energy, so to fulfil the energy demand posed by citizens of each country, their respective governments must fulfill this energy demand through other sources. The most common source of energy as of today is crude oil, therefore reducing the impacts of this commodity on the environment, as well as making it a feasible solution cost-wise is of a high importance. Thus, this paper aims to solve this aspect in the transition to a world with sustainable energy sources, whilst maintaining energy security in nations. It aims to do so by aiding policy-makers' decisions of where to import oil from,

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in order to decrease the total cost (both shipping cost and price of oil) and the pollution cost (from carbon dioxide emissions in the refining process and sulphur dioxide emissions whilst burning). As each country is attempting to transition to a world which is sustainable and environmentally friendly, it is of a high priority for all member nations of the UN to come together to adopt policies to reduce environmental impacts of their oil imports, and hence this paper tackles the problem of minimising global environmental impact and cost.

3 Problem Objective and MILP Techniques

This section discusses the two main objectives that this paper aims to solve and discusses some of the MILP techniques used in solving these problems.

3.1 Problem Objective: Single Country Optimisation

The first step towards solving the UN Sustainable Development Goal regarding clean and affordable energy, is to optimise the cost and pollution of oil imports for a single country. This will aid policy-makers in making decisions from where to import oil from, and how much oil to import from these nations. Nations would be fulfilling their responsibility in reducing their impact on the environment, whilst also maintaining cost budgets to ensure that energy demand of the nation is met. Additionally, this problem can easily be adapted and solved by adding constraints on which nations a country is a trading partner with, or changing the objective function depending on the friendliness from the nation it is dealing with. Using this as a guiding tool, policy-makers will be able to more easily justify their decision of oil imports from nations. Hence this paper provides a method to solve this problem, with room for tweaks to be made to customise the objective and constraints for individual countries.

3.2 Problem Objective: Global Optimisation

A further expansion of this problem is explored when trying to solve for an optimal cost and pollution for all countries together. This is an even more important goal as it aligns even more closely with the United Nations SDG number 7 and SDG Number 17, with all member nations uniting to work towards a common goal. This problem would use similar formulation as the first problem of policy-making for one country, however would involve minimising the cost and pollution of each country's choice of import, rather than just one country's choice of import.

3.3 Mixed Integer Linear Programming Discussion

Usually, Mixed Integer Linear Programming (MILP) is implemented to solve optimisation problems and to perform system analysis involving integer variables, and convex constraints, often integrating many different techniques such as Robust Optimization or Bi-Level Optimisation, Big M Formulation, Branch and Bound, and Multi-Objective Optimisation to solve scheduling problems and the most efficient method to transfer resources, whilst serving some physical constraints (i.e when transporting notebooks, you can not transfer half a book).

3.4 Advantages of Using Mixed Integer Linear Programming

As discussed in [Chang et al. 2004], there are many advantages of Mixed Integer Linear Programming. Hence, given that the problem posed was convex and included physical constraints (i.e number of barrels of oil transported must be an integer), the most suitable technique to use was MILP. The advantages of this technique compared to linear programming and other optimisation methods were many-fold, listed below.

1. MILP considers variables which are bound by physical constraints, and variables which are unbounded
2. The solution time of the MILP formulation is far less (computationally cheaper)
3.The power of the method is very large, as it can deal with variables in matrices of significant sizes
4.The dual and primal values prove that the solution is feasible and is the optimal solution after using the branch and bound technique

3.5 Primal and Dual Values

Primal and dual values are calculated by the optimizer to ensure that the problem is feasible. In a minimization problem, such as the one proposed, the primal is a minimization problem, and the dual is formed by using non-negative Lagrange Multipliers to add further constraints to the objective function. This turns the dual problem into a maximization problem. The solution to the primal problem is an upper bound to the solution of the dual and the dual is the lower bound to the solution of the primal. In certain convex problems which fall under constraint qualification condition, the duality gap (difference between the primal and dual values) is zero, which indicates strong duality, and gives the optimal solution to the minimization problem. The problem discussed in this paper is one such example of a problem with strong duality.

3.6 Optimizer Used

The problem is solved using the HiGHS optimizer, which is one of the powerful linear optimisers [Lubin et al. 2022] of the JuMP module, in Julia programming language as shown in Figure 2. This optimizer is most suitable for solving linear problems as this is built specifically for optimizing using linear constraints for convex optimization. By solving the primal and dual, it determines the feasibility of the solution. Moreover, the HiGHS optimizer is very good with dealing with matrices of variables, multiple constraints, and has an easily definable and adaptable objective function.

```julia
model = Model(HiGHS.Optimizer)
@variable(model, 0 <= transfer[a in exportingcountries, m in importingcountries],Int)

for i in importingcountries
    for e in exportingcountries
        if i == e
            @constraint(model,transfer[e,i] == 0)
        end
        @constraint(model,transfer[e,i] <= supply[e])
    @constraint(model,transfer[e,i] <= totaldemand[i])
    end
end
for i in importingcountries
    @constraint(model,sum(transfer[:,i]) == totaldemand[i])
end

for j in exportingcountries
    @constraint(model,sum(transfer[j,:]) <= supply[j])
end
@objective(model,Min,sum(transfer[m,n] * totalcost[m] for m in exportingcountries for n in importingcountries) +
    sum(transfer[m,n] * pollution[m] for m in exportingcountries for n in importingcountries) +
    sum(transfer.*selectiveshippingcost))
optimize!(model)
```

Figure 2: HiGHS Optimizer Code in Julia JuMP

4 Notation

$P(\alpha,\beta)$ = Price of Crude Oil (dollars per Barrel)
α = API Density
β = Sulphur Percentage
γ = Quantity of Oil per Barrel
$CO_2(\alpha)$ = Carbon Dioxide Emissions (Pounds Per Barrel)
$SO_2(\beta,\gamma)$ = Sulphur Dioxide Emissions (Pounds Per Barrel)
l = shipping distance between two countries
C (l) = cost (dollars per barrel)
k = Number of Grades of Oil per country
x = Importing country name (single country optimisation)
S = Total number of exporters of crude oil
D = Total number of importers of crude oil (global optimisation)
s = List of countries exporting oil
d = List of countries importing oil
w = Weight of Sulphur Dioxide Emissions
w_1 = Weight of Carbon Dioxide Emissions
CO2 = List of CO2 values
SO2 = List of SO2 values
p = List of price of Oil per barrel
c = Matrix of shipping costs
q = Matrix of quantities of oil being exported and imported
A = Objective function for Single Country Optimisation
B = Objective function for Global Optimisation
W = set of all whole numbers

5 Methodology

The database used was that of the United Nations' energy statistics. From this database, the oil imports and exports section were extracted, as shown in [Figure 3]. The database also consisted of the yearly consumption and production of oil, however for this problem, the export and import data was used, as many countries may have had reserves of oil from previous years, or may have had a different policy on total amount of imports and exports due to the market price of grade of oil they are producing.

Index	country or area	commodity transaction	year	unit	quantity
174206	Algeria	Conventional crude oil – imports	2014	Metric tons, thousand	294
174231	Argentina	Conventional crude oil – imports	2014	Metric tons, thousand	509
174256	Aruba	Conventional crude oil – imports	2014	Metric tons, thousand	0
174281	Australia	Conventional crude oil – imports	2014	Metric tons, thousand	21835
174306	Austria	Conventional crude oil – imports	2014	Metric tons, thousand	7510
174349	Bahrain	Conventional crude oil – imports	2014	Metric tons, thousand	10385
174374	Bangladesh	Conventional crude oil – imports	2014	Metric tons, thousand	1308
174407	Belarus	Conventional crude oil – imports	2014	Metric tons, thousand	22508
174430	Belgium	Conventional crude oil – imports	2014	Metric tons, thousand	32188
174455	Bosnia and Herzegovina	Conventional crude oil – imports	2014	Metric tons, thousand	934
174470	Brazil	Conventional crude oil – imports	2014	Metric tons, thousand	17757
174495	Bulgaria	Conventional crude oil – imports	2014	Metric tons, thousand	5103
174520	Cameroon	Conventional crude oil – imports	2014	Metric tons, thousand	1956
174534	Canada	Conventional crude oil – imports	2014	Metric tons, thousand	38614

Figure 3: United Nations data with nation wise Crude Oil Exports and Imports

5.1 Grades of Oil and Price Dependency on API Density and Sweetness - Multiple Linear Regression

Different grades of oil, determined by the API Density and the sweetness (sulphur percentage) , as shown in Figure 4, were used as metrics to distinguish the different types of oil each country produced. The data for price was determined through the methods of multiple linear regression and extrapolation.

The price of 12 different grades of oil, obtained from an oil website were regressed against sulphur percentage and API Density, using the least sum of squares criterion to help create a metric for the dependence of price on API Density and sulphur percentage.

These 12 grades of oil were chosen as they were the most stable grades of oil, and the political factors, as well as market characteristics affecting these grades were minimal. Hence, from this extrapolation was used to predict the fair prices of other grades of oil using the coefficients of API Density, sulphur and a constant term, as shown in (1).

$$P(\alpha,\beta) = 135.57 - 0.66\alpha - 4.98\beta. \tag{1}$$

Crude	API	Sulfur	Source
Agbami	47.2	0.05	Nigeria
Akpo	45.8	0.07	Nigeria
Al Shaeen	30.3	1.90	Qatar
Amenam	37.0	0.17	Nigeria
Amna	37.0	0.17	Libya
ANS	31.4	0.96	US
Arab Extra Light	40.0	1.09	Saudi Arabia
Arab Heavy	28.0	2.80	Saudi Arabia
Arab Light	33.0	1.77	Saudi Arabia
Arab Medium	31.0	2.55	Saudi Arabia
Arab Super Light	51.0	0.09	Saudi Arabia
Ardjuna	37.0	0.09	Indonesia
Attaka	43.0	0.09	Indonesia
Azeri Light	34.9	0.55	Azerbaijan
Bach Ho	39.0	0.04	Vietnam
Bakken	42.1	0.18	US
Banoco	31.8	2.45	Bahrain

Figure 4: Crude Oil Grades with the different API Density and Sulfur Percentage. Source: McKinsey Energy

- Use the following equation to calculate SO_2 emissions from fuel usage: SO_2 = S x F x 2

  Where: SO_2 = sulfur dioxide emissions for the quantity of fuel burned, tons
  S = sulfur content of the fuel burned (fraction by weight)
  F = quantity of fuel burned, based on quantity measurement, tons
  2 = pounds of sulfur dioxide per pound of sulfur

Figure 5: Formula for SO_2 emissions using Sulphur percentage and quantity of oil burned. Source: Oregon Government

5.2 Approximating Sulphur Dioxide Emissions from Burning of Oil

In this section, pollution from sulphur dioxide will be obtained using a formula for sulphur dioxide produced from burning oil with a certain sulphur percentage as shown in Figure 5.

This is one of the two metrics used to model the impact of different grades of oil on the environment.

$$SO_2(\beta,\gamma) = 2\beta\gamma. \tag{2}$$

5.3 Approximating Carbon Dioxide Emissions in Refining of Oil

A metric shown in [Brandt 2011] and in Figure 6 is used to estimate the carbon dioxide emissions produced in the refinery of oil given a certain API Density using a method of linear regression, with R squared value of 0.86. Using the coefficients given as an approximation, the CO2 emissions produced in the refining of the crude oil can be calculated, as shown in (3) and used in combination with SO2 emissions to serve as a metric for pollution caused by each barrel of oil.

$$CO_2(\alpha) = 86.39 - 0.77\alpha. \quad (3)$$

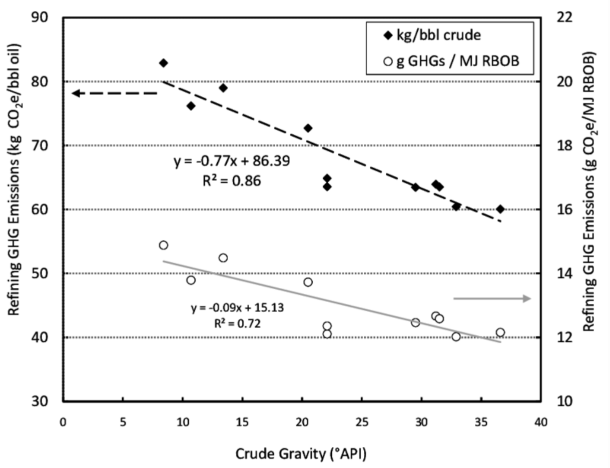

Figure 6: Approximation of CO_2 emissions from refining of crude oil using linear regression against API Density. [Brandt 2011]

5.4 Approximating Oil Statistics For Countries with Multiple Grades of Oil

Some countries such as the United States of America had multiple different grades of oil, and there was no data on the distribution of the percentages of oil, hence an assumption was made that there was an equal distribution of each grade of oil being exported. Hence, the price, SO2 emissions and CO2 emissions of each country were approximated as the mean of all grades of oil from that country.

$$\frac{1}{k}\sum_{n=1}^{k} CO2_n \quad (4)$$

$$\frac{1}{k}\sum_{n=1}^{k} SO2_n \quad (5)$$

$$\frac{1}{k}\sum_{n=1}^{k} p_n \quad (6)$$

5.5 Approximating Oil Statistics For Countries with No Data about Grades of Oil

Some countries did not have any data regarding the API density and sulfur percentage of the oil they were exporting. Hence, given the fact that brent crude oil is the most common type of oil and the fact that these countries don't have their own oil grade, it can be inferred that these countries have oil of similar API Density and sulfur percentage to brent crude oil, so it can be approximated to have the same price, CO_2 emissions and SO_2 emissions as brent crude oil. Hence, these statistics are now used for the optimisation.

5.6 Approximating Shipping Cost using Distance from Ports as an Indicator

Another statistic which had to be calculated was shipping cost from each country to the other country in terms of cost per barrel of oil. [Bertoli et al. 2016] approximated the distance from each country to all other countries via sea in nautical miles by assuming it to be the distance between two major parts of both countries and then calculating the distance between these two ports. From expert discussions on various websites, it was approximated that the cost of transporting oil was 1 dollar per barrel of oil per 1000 nautical miles. Hence, the data from the CERDI database was used to approximate cost of transport as a function of sea distance through the formula below (7). Thus, a matrix for cost of transportation per barrel of oil from each country to the other countries was obtained.

$$C(l) = \frac{l}{1000}. \quad (7)$$

5.7 Objective Function Discussion

Lastly, for the objective function it is very important to give suitable weights to the two different metrics for the objective function to be minimised. On one hand, countries want to reduce cost (shipping cost + cost of oil per tonne) whilst they also want to reduce the "cost" of pollution (SO_2 emissions + CO_2 emissions per tonne). However, given which objective they wish to prioritise, they will give this a higher weightage in the objective function. This weightage can be changed in the objective function by adding different coefficients to the different objectives to prioritise one objective over the other. There are a few different ways to choose these weights.

5.7.1 *Weights using Carbon Tax Policy*

In this method, the weight for the pollution metric could be calculated using carbon tax and sulphur dioxide removal costs to approximate carbon dioxide and sulphur dioxide as costs. Then the objective of reducing cost and pollution could be added given that the units are same. The value used as a weight for the carbon tax can be found in [OECD 2018], and the value for sulphur dioxide removal costs can be found in [Burtraw 2000].

5.7.2 *Weights using Preference*

In this method, United Nations could use statistics regarding the cost of CO_2 and SO_2 emissions to society through the impact on the healthcare system, wildlife and air quality to design a suitable metric to give weights, which could be added to minimise the total objective function.

6 Problem Formulation and Constraints

In this section, we will be discussing the formulation of the problem and the constraints, as well as objective functions for both of the different objectives

6.1 Goal 1: Problem Formulation for One Country's Policy-Making

This section will discuss the formulation of the constraints and objective for the single country optimisation problem.

6.1.1 *Constraints for Goal 1*

This section contains 2 constraints for the single country optimisation.
1. The first constraint shows that the quantity transfer of oil from one country to another must be a whole number (positive integer value), and it must be less than the total supply of oil from the exporting country, as shown below.
2. The second constraint shows that the sum of all imports of the country, must be equal to the total demand of that country, as shown in (8)

$$\forall i \in s, 0 \leq q_i^x \leq s^i, \exists q_i^x \in W$$

$$\sum_{i=1}^{S} q_i^x = d^x \tag{8}$$

6.1.2 *Objective Function for Goal 1*

This section highlights the objective function for the single country optimisation, which can be simplified by breaking it down into three parts, as shown below.

Part 1: (Price of oil + Shipping cost of oil) × Quantity Transfer of Oil
Part 2: Weight of SO_2 × Quantity Transfer of oil × SO_2 emissions
Part 3: Weight of CO_2 × Quantity Transfer of oil × CO_2 emissions

These three parts can be added together as they are in the same units, as in (9) and (10). As discussed in the introduction section, the objective function can be tweaked easily, to give certain importance to geopolitical situations and certain trends.

The minimisation of this objective function is solved by the HiGHS optimizer.

$$\min_{i \in s} \quad A \tag{9}$$

$$A = \sum_{i=1}^{S} (q_i^x \times (p_i + c_i^x) + w(q_i^x)((SO_2)_i) + w_1(q_i^x)((CO_2)_i)) \tag{10}$$

6.2 Goal 2: Problem Formulation for United Nation Policy-Making

This objective aims to optimise the total cost and pollution of all United Nations members collectively.

6.2.1 *Constraints for Goal 2*

There are 3 major constraints in this goal.

1. The sum of quantity of oil transferred from one nation to all other nations must be less than or equal to the total amount of supply of oil from that one nation

2. The sum of quantity of oil transferred from all nations to one nation must be equal to the total demand of oil from that one nation.

3. The quantity of oil transferred must always be a whole number (positive integer).

$$\forall i \in s, 0 \leq \sum_{n=1}^{D} q_i^n \leq s^i \tag{11}$$

$$\forall n \in d, 0 \leq \sum_{i=1}^{S} q_i^n = d^n \tag{12}$$

$\forall a \in s, \forall b \in d, \exists q_a^b \in W$

6.2.2 *Objective Function for Goal 2*

The objective function can once again be split into 3 parts.
Part 1: (Price of oil + Shipping cost of oil) × Quantity Transfer of Oil
Part 2: Weight of SO_2 × Quantity Transfer of oil × SO_2 emissions
Part 3: Weight of CO_2 × Quantity Transfer of oil × CO_2 emissions

These three parts are added together to form the objective function, as shown below. The only difference in this objective function and the previous one is that the objective function has multiple demanding nations, hence will have to be summed over all the demanding nations as well.

Once again, depending on certain external factors in certain countries, tweaks can be made to this objective function, but this serves as the base function, including all the essential costs.

$$\min_{i \in s, n \in d} \quad B \tag{13}$$

$$B = \sum_{i=1}^{S} \sum_{n=1}^{D} (q_i^n (p_i + c_i^n) + w(q_i^n)((SO_2)_i) + w_1(q_i^n)((CO_2)_i)) \tag{14}$$

7 Results

In this section, the results of the optimization of the two objective functions will be reviewed and analysed, whilst key topics such as game theory, choosing weights and the usage of integer variables will be discussed.

7.1 Objective 1: Minimising Cost and Pollution from Crude Oil to a Member Nation

In this section, we will discuss the results from various countries, the importance of shipping cost on the results and the

Country Name	Barrels of Oil
Nigeria	756294
Norway	469647
Kazakhstan	223919
Algeria	172621
Equatorial Guinea	99037
Malaysia	83796
Australia	81355
Gabon	81297
Gabon	79105
Vietnam	68212
Ghana	39480
Chad	36940
Denmark	35176
Argentina	14506
Sudan	13106
Papua New Guinea	5891

Table 1: Result of China's Optimisation

Country Name	Barrels of Oil
Norway	469647
Algeria	172621
Nigeria	108368
Equatorial Guinea	99037
Malaysia	83796
Australia	81355
Gabon	81297
Vietnam	68212
Ghana	39480
Chad	36940
Denmark	35176
Argentina	14506
Sudan	13106
Papua New Guinea	5891

Table 2: Results of India's Optimisation

implementation of carbon tax and sulphur dioxide replacement as a method to give a weight to the pollution index in the objective function, and the results from this.

7.1.1 *Discussion of Results from India and China*

As can be seen from the sample table above, to ensure the optimal policy for oil imports from other countries, the constraints set in the problem formulation must be met and the objective function must be minimal. The results can be seen to be very similar, as shown in Table 1 and Table 2 given the fact that most features remain constant when changing from India to China. For example, the CO_2 emissions, SO_2 emissions and Price of Oil per barrel stay the same. The only thing which changes is the shipping cost, and the quantity of oil being demanded for import. Additionally, given that China and India are very close together in terms of distance, shipping cost differences should be minimal.

Country Exporting	Country Importing	Barrels of Oil
Albania	Aruba	2047
Albania	Bosnia and Herzegovina	224141
Albania	Belarus	6504
Argentina	Switzerland	13621
Australia	Brazil	24614
Belarus	Belgium	11852
Brazil	Austria	151851
China	Belgium	4399
Czechia	Austria	197
Equatorial Guinea	Brazil	99037

Table 3: Results of Global Optimisation

7.1.2 *Dependence of Results on Distance - Shipping Cost*

If a third country, say the United States is chosen, it may give very different results to these two countries noted above, given the fact that the shipping distance from other nations varies largely. Recall that the shipping cost between two countries can be calculated using (7). Furthermore, as discussed in the methodology, if coefficients are added to each objective to weight one more than the other, this will obviously change the solution. For example, if the environmental pollution is given a coefficient of 10 times greater than cost, then results will be skewed towards importing from countries which have the most clean oil, and if cost is given higher coefficient, then results will be skewed towards obtaining the cheapest oil possible, without considering the impact on the environment as significantly.

7.1.3 *Discussion of Carbon Tax and Sulfur Dioxide Cost to choose weights of pollution*

A solution to choose the optimal weights, would be to use the carbon credit or carbon tax system as a metric to estimate the impact of Carbon Dioxide emissions on each nation, and to estimate the weight of sulfur dioxide as the cost of reducing sulfur dioxide emissions per tonne. These weights give a good estimate of the "cost" of carbon emissions and hence the pollution metric and cost of shipping and oil per barrel can be added together as they have the same units.

7.2 Objective 2: Minimising Global Cost and Pollution from Crude Oil as a United Nations Policy Maker

In this section we will discuss the results of the global optimisation problem and the implementation of game theory, as well as the use of integer vs decimal variables

7.2.1 *Discussion of How to Interpret Results*

The table above, as shown in Table 3 is a sample of the solution for optimising the transfer of oil from one country to another. The table can be read by seeing that each column represents the total amount of oil the importing country takes from each of the exporting countries. For example, in the first column, as shown in Table 3 it can be seen that Aruba imports 2047 barrels of crude oil from Albania and no oil from the remaining countries in this sample.

7.2.2 *Discussion of Integers vs Decimal Variables*

Another factor to discuss is the implementation of integer variables vs relaxing the constraint of integers for the JuMP function. As can be seen from the below summary, the solve time for integer constraints is double the time for decimal constraints. In addition, both strategies are just indicative measures for the imports of a country. Whilst in reality, a country is unable to import a fraction of a barrel of oil, it may not practically be able to import an integer which the algorithm suggested, due to the fact that a ship transporting the oil may only be able to facilitate a certain amount of barrels of oil, and the fixed cost of running another ship is too large to justify importing oil from that country. When changing from decimal to integer, the objective function is changed by less than 0.1 percent, hence both solutions are equally useful indicative measures of the optimal supply. The solve time for a MILP formulation with integer constraints is more than that without any integer constraints, as discussed in [Leiserson and Saxe 1988]. Given the fact that both solutions are similar, and that this is only an indicative measure, the decimal solution could also be used, as it is easier for the HiGHS optimizer to solve, and is computationally cheaper.

8 Conclusions and Scope for Future Work

8.1 Discussion of Game Theory - Nash Equilibrium and Shared Utility

The optimizer in the United Nations optimisation problem, solves for a global optimum solution, which is a huge step for the members of the United Nations, by keeping the cost and pollution to a minimum. However, to ensure that the cost is distributed in an equitable manner, it is important to apply the Nash equilibrium, or another constraint to penalise large difference in cost and pollution between 2 or more countries. This will be a further step towards making the world more equal, and though the objective cost may be fractionally higher, all countries would think of this as a fair solution, and policy-makers would be contented to implement such a strategy. A different approach to solving the problem of large disparity in objective functions between 2 or more countries, is to use the concept of shared utility. Countries who have received a lower objective cost and pollution, could share a certain percentage of their payoff with the less advantaged nations, to ensure that all nations are satisfied with this solution, and that the global optimum indeed is the best solution for all nations.

The global optimum, however, itself is a large step towards the goal of achieving clean and affordable energy, as all members of the United Nations collectively are reducing the pollution to the environment and it is made sustainable by reducing total costs, and hence making it a long term feasible solution.

8.2 Difference in Objective Value for each country for Global Optimisation vs Optimisation for a single country

This section adds to the previous section in finding a unique solution to provide a country an incentive or disincentive to ensure that each country follows the global solution provided. Either countries can be given a payoff, to use the concept of shared utility to help and make all stakeholders happier in the process or the UN can implement a tariff or levy for all countries who do not follow the policy, which increases their total cost beyond the optimum being provided in the global optimisation.

As stated previously to make it the most fair distribution rather than most optimal, the Nash Equilibrium should be found or there should be a constraint to squeeze the difference between objective costs for each country. However, in this paper, the global optimum solution is being found, as this shows how each country should proceed to buy oil. The distribution of profit (shared utility) is a further step to make this more equitable, however the total global cost stays the same.

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References

Alberizzi, J. C., M. Rossi, and M. Renzi (2020). "A MILP algorithm for the optimal sizing of an off-grid hybrid renewable energy system in South Tyrol". *Energy Reports* 6, pages 21–26.

Archer, R., G. Nates, S. Donovan, and H. Waterer (2011). "Wind turbine interference in a wind farm layout optimization mixed integer linear programming model". *Wind Engineering* 35(2), pages 165–175.

Bertoli, S., M. Goujon, and O. Santoni (2016). "The CERDI-seadistance database".

Brandt, A. R. (2011). "Upstream greenhouse gas (GHG) emissions from Canadian oil sands as a feedstock for European refineries. Department of Energy Resources Engineering". *Stanford, CA, January* 18, page 51.

Burtraw, D. (2000). *Innovation under the tradable sulfur dioxide emission permits program in the US electricity sector*. Technical report.

Chang, G. W., Y. Tsai, C. Lai, and J. Chung (2004). "A practical mixed integer linear programming based approach for unit commitment". In: *IEEE Power Engineering Society General Meeting, 2004.* IEEE, pages 221–225.

Cosic, A., M. Stadler, M. Mansoor, and M. Zellinger (2021). "Mixed-integer linear programming based optimization strategies for renewable energy communities". *Energy* 237, page 121559.

Kantor, I., J.-L. Robineau, H. Bütün, and F. Marechal (2020). "A mixed-integer linear programming formulation for optimizing multi-scale material and energy integration". *Frontiers in Energy Research* 8, page 49.

Kuo, J. Y., D. A. Romero, J. C. Beck, and C. H. Amon (2016). "Wind farm layout optimization on complex terrains–Integrating a CFD wake model with mixed-integer programming". *Applied Energy* 178, pages 404–414.

Leiserson, C. E. and J. B. Saxe (1988). "A mixed-integer linear programming problem which is efficiently solvable". *Journal of Algorithms* 9(1), pages 114–128.

Lubin, M., O. Dowson, J. D. Garcia, J. Huchette, B. Legat, and J. P. Vielma (2022). "JuMP 1.0". *arXiv preprint arXiv:2206.03866*.

Methlouthi, I., M. Gendreau, and J.-Y. Potvin (2016). *Mixed integer programming for a multi-attribute technician routing and scheduling problem.* CIRRELT, Centre interuniversitaire de recherche sur les réseaux d'entreprise …

Morais, H., P. Kádár, P. Faria, Z. A. Vale, and H. Khodr (2010). "Optimal scheduling of a renewable micro-grid in an isolated load area using mixed-integer linear programming". *Renewable Energy* 35(1), pages 151–156.

OECD (2018). "Effective carbon rates 2018: Pricing carbon emissions through taxes and emissions trading". *OECD*.

Ren, H. and W. Gao (2010). "A MILP model for integrated plan and evaluation of distributed energy systems". *Applied energy* 87(3), pages 1001–1014.